Engineering Notes

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Remarks on Thin Airfoil Theory

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IN the thin airfoil theory (thickness problem) one needs to solve[1]

$$\frac{u}{U}=\frac{1}{\pi}\int_{-1}^{1}\frac{T'(x)}{x-\xi}\mathrm{d}x \qquad (1)$$

where u is the perturbation velocity in the x-direction on the airfoil surface, U the freestream velocity, x,ξ are as shown in Fig. 1, $T(x)$ is the airfoil thickness function, and prime denotes differentiation with respect to the function argument. It is usually more convenient to work with Eq. (1) after the transformation

$$x=-\cos\theta \qquad \xi=-\cos\phi \qquad (2)$$

so that Eq. (1) becomes

$$\frac{u}{U}=-\frac{1}{\pi}\int_{0}^{\pi}\frac{T'(\theta)\mathrm{d}\theta}{\cos\theta-\cos\phi} \qquad (3)$$

Eq. (3) may be easily solved if $T(\theta)$ is expanded in a trigonometric series

$$T(\theta)=\sum_{n=1}^{\infty}A_n\sin n\theta+\sum_{m=1}^{\infty}B_m\cos m\theta+B_0 \qquad (4)$$

Upon substituting Eq. (4) into Eq. (3), the sin terms in $T(\theta)$ give rise to the famous Glauert integral

$$\int_0^{\pi}\frac{\cos m\theta}{\cos\theta-\cos\phi}\mathrm{d}\theta=\pi\frac{\sin m\phi}{\sin\phi} \qquad (5)$$

and for the cos terms, as far as the author is aware, no closed form solution has been given. It may be because the literature seems to imply that the thickness distributions are adequately described by the sin terms alone.[1-3] This is not true as may be seen from Table 1 where the biconvex airfoil is represented by cos terms. As a general rule, $T(\theta)$ will contain cos terms whenever a sharp edge is present and sin terms whenever a rounded edge is present. The constants A_n, B_m must satisfy the following conditions:

1) For airfoils closed at both ends

$$\sum_{m=0}^{\infty}B_{2m}=0 \qquad \sum_{m=0}^{\infty}B_{2m+1}=0 \qquad (6)$$

2) For rounded edges

$$\left(\sum_{n=1}^{\infty}n\,A_n\right)^2=r_L \qquad \left(\sum_{n=1}^{\infty}(-1)^n n\,A_n\right)^2=r_T \qquad (7)$$

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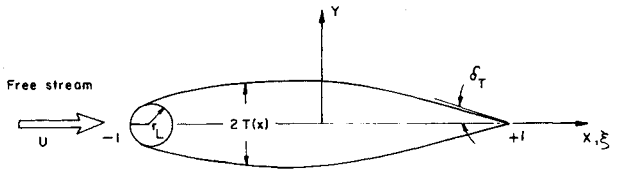

Fig. 1 Airfoil nomenclature.

where r_L and r_T are the leading-edge and trailing-edge radii, respectively.

3) For sharp edges

$$\sum_{n+1}^{\infty}n\,A_n=0 \qquad \sum_{m=1}^{\infty}m^2B_m=-\tan|\delta_L|$$

$$\sum_{n=1}^{\infty}(-1)^n n\,A_n=0 \qquad \sum_{m=1}^{\infty}(-1)^m m^2B_m=\tan|\delta_T| \qquad (8)$$

where δ_L and δ_T are the leading-edge and trailing-edge angles respectively (see Fig. 1).

Now to complete the solution of Eq. (3) for $T(\theta)$ given by Eq. (4) we need to evaluate the integrals

$$\int_0^{\pi}\frac{\sin n\theta}{\cos\theta-\cos\phi}\mathrm{d}\theta$$

This is done as follows

$$J_n\equiv\int_0^{\pi}\frac{\sin n\theta\mathrm{d}\theta}{\cos\theta-\cos\phi}=\int_0^{\pi}\frac{\sin n\theta-\sin n\phi}{\cos\theta-\cos\phi}\mathrm{d}\theta \qquad (9)$$

From the trigonometric relation

$$\sin(n+1)\theta-\sin(n+1)\phi+\sin(n-1)\theta-\sin(n-1)\phi$$
$$=2\sin n\theta(\cos\theta-\cos\phi)+2\cos\phi(\sin n\theta-\sin n\phi) \qquad (10)$$

we have

$$J_{n+1}+J_{n-1}=2\cos\phi\,J_n-(2/n)[1-(-1)^n] \qquad (11)$$

When n is even

$$J_{n+1}+J_{n-1}=2\cos\phi J_n \qquad (12)$$

which has the solution

$$J_n=A\sin n\phi+B\cos n\phi \qquad (13)$$

Table 1 Some airfoil representations

Airfoil	$T(\theta)$	Thickness/chord	Leading-edge radius
Elliptic	$\epsilon\sin\theta$	ϵ	ϵ^2
Joukowski	$\epsilon(\sin\theta+½\sin 2\theta)$	$\frac{3\sqrt{3}}{4}\epsilon$	$4\epsilon^2$
Biconvex	$\frac{\epsilon}{2}(1-\cos 2\theta)$	ϵ	0

It is fairly straightforward to evaluate J_0 and J_1. These are

$$J_0 = 0 \tag{14}$$

$$J_1 = \int_0^\pi \frac{\sin\theta - \sin\phi}{\cos\theta - \cos\phi} d\theta = \int_0^\pi \cot\left(\frac{\theta+\phi}{2}\right) d\theta$$

$$= 2 \log \cot(\phi/2) \tag{15}$$

Hence we conclude from Eq. (13) that

$$A = 2 \log \cot\phi/2 - 4/\sin 2\phi$$

$$B = 0 \tag{16}$$

Thus

$$J_n = 2\frac{\sin n\phi}{\sin\phi} \log \cot(\phi/2) - 4\frac{\sin n\phi}{\sin 2\phi} \quad (n \text{ even}) \tag{17}$$

When n is odd

$$J_{n+1} + J_{n-1} = 2\cos\phi \; J_n - 4/n \tag{18}$$

The left-hand side has even indices, hence using Eq. (17) we have

$$J_n = 2\frac{\sin n\phi}{\sin\phi} \log \cot(\phi/2) - 4\frac{\sin n\phi}{\sin 2\phi} + \frac{2}{n \cos\phi} \quad (n \text{ odd}) \tag{19}$$

Combining Eqs. (17) and (19) we have

$$\int_0^\pi \frac{\sin n\theta}{\cos\theta - \cos\phi} d\theta = 2\frac{\sin n\phi}{\sin\phi} \log \cot\frac{\phi}{2} - 4\frac{\sin n\phi}{\sin 2\phi}$$

$$+ \left[1 - (-1)^n\right] \frac{1}{n \cos\phi} \tag{20}$$

With Eqs. (5) and (20), the solution of Eq. (3) is complete.

References

[1]Van Dyke, M. D., "Second Order Subsonic Airfoil Theory Including Edge Effects," NACA Rept. 1274, 1956.

[2]Riegels, F. W., *Aerofoil Sections*, Butterworths, London, 1961.

[3]Milne-Thompson, L. M., *Theoretical Aerodynamics*, MacMillan & Co. Ltd., London, 1958.

Technical Comments

Comment on "An Inverse Boundary-Layer Method for Compressible Laminar and Turbulent Boundary-Layers"

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THE inverse boundary-layer calculation is one of the classic goals of aerodynamics and the present paper is worthy of the subject. There are only a few minor points requiring clarification. The first point is the matter of turbulent separation prediction which the author feels can be adequately handled by the Cebeci/Smith boundary-layer computation down to zero skin friction; the experimental basis for this statement is said to be presented in Ref. 1. The second point is the author's statement that the capability does not exist today for calculating partially separating flows in two dimensions.

It is a fortuitous coincidence that we have recently presented a methodology[2] for the prediction of the pressure distribution on two-dimensional airfoils with massive turbulent separation; excellent experimental agreement was achieved for the NACA 63-018 at 18° angle of attack, for the NACA 65, 2-421 at 20° and for the NASA GA(W) – 1 at 21°. Our algorithm employed the Cebeci/Smith boundary-layer method, and it was found that it did not yield good results when using the zero skin-friction separation criterion as compared to the test data and to the algorithm with the Goldschmied[3] maximum pressure-recovery criterion.

It is also pointed out in Ref. 2 that meaningful separation prediction requires two parameters, i.e., chordwise location *and* pressure coefficient, while the evidence of Ref. 1 is concerned only with chordwise location. This can be further illustrated in Fig. 8 of the present paper: with the direct method, C_f does not go to zero (no separation) if the correct pressure distribution is employed, while with the inverse method a steeper pressure distribution is required to make C_f reach zero (separation) at the correct location.

The Cebeci method does not allow to predict *both* the correct skin-friction *and* the correct pressure coefficient when separation is present. If the data of Fig. 8 are used to verify directly the separation criterion of Ref. 3, since both skin-friction coefficients and pressure distribution were measured, it is found that good agreement exists for both pressure recovery and axial separation location.

Using the experimental plots of Strickland and Simpson,[4] I locate the minimum pressure point at $X = 65$ in. with $U = 85$ fps (Fig. 3-3, p. 30) and the corresponding turbulent skin-friction coefficient C_f between 0.0032 and 0.0034 (Fig. 3-30, p. 66), yielding a predicted maximum pressure-recovery coefficient C_{ps} between 0.64 and 0.68. The minimum velocity shown in Fig. 3-3 is $U = 51$ fps, yielding the experimental $C_{ps} = 1 - (51/85)^2 = 0.64$.

From Fig. 3-30, experimental separation is located between $X = 140$ in. and 150 in.; intersecting the inviscid attached pressure distribution at $C_{ps} = 0.64$ and 0.68, the axial separation location is obtained between $X = 140$ in. and 146 in.

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Index category: Boundary Layers and Convective Heat Transfer – Turbulent.

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References

[1]Cebeci, T., Mosinskis, G.J., and Smith, A.M.O., "Calculation of Separation Points in Turbulent Flows," *Journal of Aircraft*, Vol. 9, Sept. 1972, pp. 618-624.

[2]Farn, C.L.S., Goldschmied, F.R., and Whirlow, D.K., "Pressure Distribution Prediction for Two-Dimensional Hydrofoils with Massive Turbulent Separation," *Journal of Hydronautics*, Vol. 10, July 1976, pp. 95-101.

[3]Goldschmied, F.R., "An Approach to the Turbulent Incompressible Separation Under Adverse Pressure Gradients," *Journal of Aircraft*, Vol. 2, March/April 1965, pp. 108-115.

[4]Strickland, J.H. and Simpson, R.L., "The Separating Boundary-Layer: An Experimental Study of an Airfoil Type Flow," Southern Methodist University, Dallas, Texas, Technical Report WT-2, Aug. 1973.